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Geometric Optimization for Cost-effective Paneling of Architectural Freeform Skins

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Abstract: Paneling curved architectural skins often require rationalization prior to fabrication or the use of fabrication-aware design tools. We present optimization methods based on concepts from classical and discrete differential geometry, which can provide a significant cost reduction. After a short overview of early contributions and their use in real projects, we turn to recent work. This includes the use of spherical panels, cost reductions achieved via surfaces with a relation between principal curvatures, paneling with cold bent glass, and a dramatic reduction of the number of required molds when working with sheet metal or similarly behaving materials. Finally, we present a novel fabrication-aware interactive computational design tool.

1 Introduction

Pioneering research on computational support for the realization of architectural freeform structures, e.g. by Glymph et al. [7] and Shelden [17], sparked a remarkable stream of work on Architectural Geometry and a rapid development of key techniques for the design and fabrication of freeform architecture. This research lies at the interface of Applied Mathematics (mainly discrete differential geometry and numerical optimization), Geometry Processing, and Computational Architectural Design and Engineering (see e.g. the survey [14]).

The present paper tackles an important sub-problem of Architectural Geometry, namely paneling geometrically complex architectural surfaces. In Section 2, we review some essential early contributions and point to realized architectural projects. In those projects, Evolute GmbH in Vienna has been acting as a geometry consultant and link between the state of the art in research and construction practice.

The major part presents some computational technologies that emerged in the past few years and so far have only been realized in small-scale prototypical models. We distinguish between methods where the panels are assumed to be rigid (Section 3) and panels that are flexible to some degree, even after being possibly shaped over a mold (Section 4). We conclude in Section 5 with a brief description of very recent work on interactive computational design tools that shall be made available to the community in the very near future.

2 Early paneling solutions and their applications

Flat panels. Flat panels are the simplest ones to deal with, but even that case is not as easy as one may think. The most straightforward way of using triangular panels shifts the problems from the panels to the supporting structure, since, in a triangle mesh, typically six beams meet at a node. For freeform geometries, there will always be torsion in the nodes, which are then geometrically quite complicated objects. While the support structure becomes simpler for quadrilateral panels, one has limitations on the layout related to the curvature behavior of the reference surface. Natural relations to discrete differential geometry help to solve the underlying geometric optimization problems. We refer to [8, 18, 14] for more details. Projects, where such optimization algorithms have been used, include the Museum of Islamic Art in the Louvre (supporting hybrid mesh of triangular and planar quads) and the roof of the Chadstone Shopping Center in Melbourne (planar quad panels).

Paneling algorithm of Eigensatz et al. [5]. Given a freeform surface that represents the architectural skin and an initial panel boundary layout, the goal is to attach a panel per patch of this layout so that the overall solution fits the available budget and the aesthetic expectation of the designer. Quality criteria include gaps between adjacent panels that can be hidden in the seams and angles between normals of panels at common seams. Clearly, the smaller these values are, the smoother the overall appearance of the skin will be. One works with different panel types and a user-adjustable fabrication cost per type, the cheapest being flat and the most expensive being those panels requiring a custom-made mold. Even mold re-use has been included. Important degrees of freedom lie in the controllable deviation from the given reference geometry. The algorithm delivers a family of solutions, starting with the most cost-effective panelization (at the price of lower smoothness) and progressing to smoother ones at a higher cost. Examples of the use of this algorithm in real projects include the Eiffel Tower Pavilions [1] and the stadium Arena Corinthians in São Paulo [16] (Fig. 1).

3 Recent progress on the effective use of rigid panels

Spherical panels. The availability of spherical glass panels (see e.g. [15]) motivated research on the use of spherical rather than flat panels. Even if one restricts to a small number of sphere radii, one can achieve an improvement in the visual appearance over flat panels, while keeping other essential aspects, such as torsion-free supporting structures. The geometric fundamentals for such paneling solutions are found in a very recent contribution on meshes with spherical faces [10]. It uses methods from the so-called Möbius geometry of spheres and geometric optimization to provide a systematic study of the various types of meshes with spherical faces and to address key aspects relevant for paneling. These include surface smoothness, the geometry of support structures, and the usage of a limited number of sphere

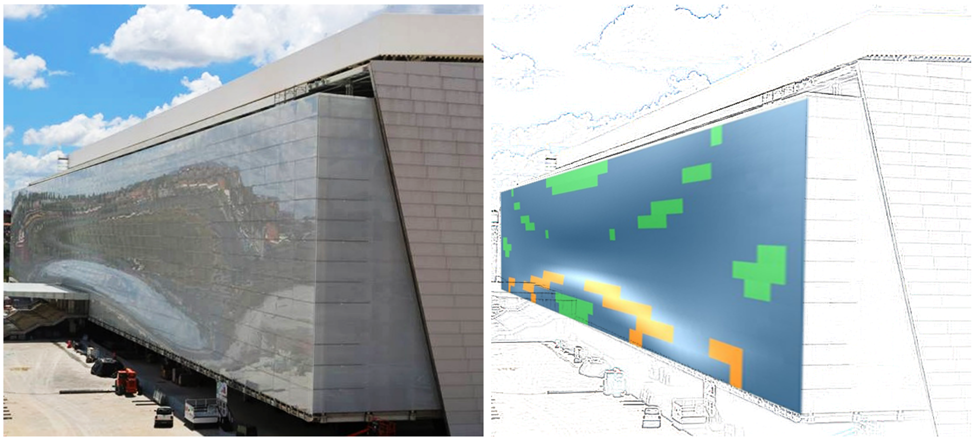

Figure 1: The west facade of the Arena Corinthians in São Paulo (architect: Anibal Coutinho) and the cost-effective partitioning into different panel types (right) [16]. Most panels (blue) are parts of right circular cylinders and can be produced by special machines. Green panels are flat; yellow panels are fabricated with a custom-made mold.

radii. As a by-product of a study of certain deployable structures, Liu et al. [11] presented surface panelizations with spherical panels of fixed radius and a torsion-free support structure. The underlying reference surfaces are so-called hyperbolic linear Weingarten surfaces, characterized by a linear relation $aK + bH + c = 0$ between Gaussian curvature K and mean curvature H under the constraint $b^2 - 4ac < 0$. There, the panel boundaries are aligned with principal curvature directions. It is not surprising that the best results are obtained in positively curved areas (see Fig. 2, right). This leads us naturally to the next topic.

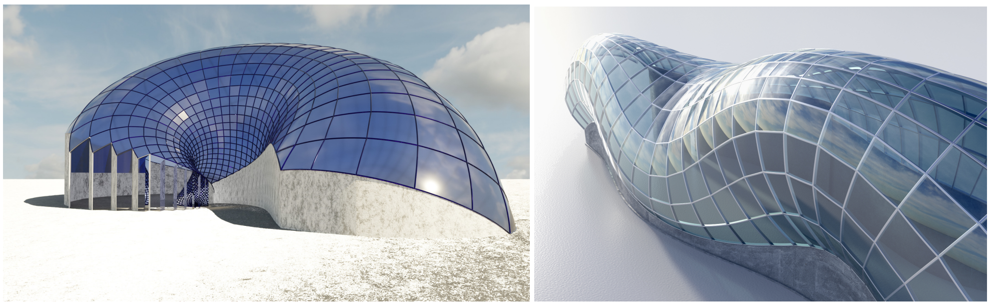

Figure 2: Left: Paneling with sphere panels over a circular mesh (quad mesh in which each face possesses a circumcircle) [10]. Right: Sphere panels of constant radius on a design surface, which exhibits a linear relation between Gaussian and mean curvature [11].

Design surface approximation by Weingarten surfaces. The two principal curvatures κ_1, κ_2 of a surface at a given point completely determine the shape of a local second-order approximation of a surface. Since architectural surfaces usually do not exhibit rapid changes of curvature, one can expect that this local approximation is already a good initial shape for a panel. If we have a surface which has only a one-parameter family of such local approximations, there must be a functional relation $f(\kappa_1, \kappa_2) = 0$ between the principal curvatures, or equivalently, a relation $F(K, H) = 0$ between Gaussian curvature $K = \kappa_1\kappa_2$ and mean curvature $H = (\kappa_1 + \kappa_2)/2$. Surfaces with such a relation are called Weingarten surfaces. For them, one can expect that panels along curves of constant K (or equivalently constant H) can be formed over the same mold. In a grid of N^2 panels, this should reduce the number of different molds to the order N. That initial idea could be verified in computational experiments [12, 13]. Pellis et al. [12] went even further and developed an algorithm that approximates a given design surface by a Weingarten surface, where the functional relation is a result of optimization. Fig. 3 shows an example.

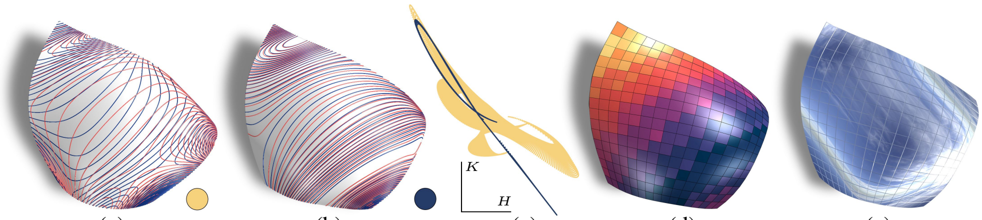

Figure 3: Approximation of a given surface by a Weingarten surface to an unknown curvature relation according to [12]. From left to right: The original surface (a) is not a Weingarten surface, since isolines of Gaussian curvature K and mean curvature H are not aligned. After optimization (b), these curves are aligned, characterizing a Weingarten surface. (c) shows the action of optimization on the (H, K)-diagram. On the original surface, curvature pairs (H, K) form a region (yellow), which contracts to a curve during optimization. For the optimized surface, the same mold (same color) can be used for panels that are roughly along the same curvature-isoline (d). The appearance of this panelization (e) is visually smooth.

4 Advantages provided by non-rigid panels

Cold-bent glass panels. As a cost-effective alternative to curved glass produced by hot bending via molds, cold-bending of glass has been explored [4]. This comes with advantages of higher optical and geometric quality and possibilities regarding printing and layering and the use of partly tempered or toughened glass, but imposes big challenges on computational design. It turned out that even the most advanced and performance-optimized simulation algorithms are insufficient in a near-interactive design environment. Here, deep learning comes to the rescue: Based on more than a million simulation results, a neural network has been trained to predict the shape and maximum stress of a glass panel being attached to a given boundary [6]. This network is then integrated into a near-interactive design tool, which allows one to design or modify freeform skins and panel layouts for optimal usage of cold-bent glass panels (see Fig. 4).

Panels from isometrically deforming material. Panels from certain materials, such as sheet metal, are still flexible after being formed over a mold. This deformation is largely isometric in the geometric sense. An effective method for the computational treatment of isometric

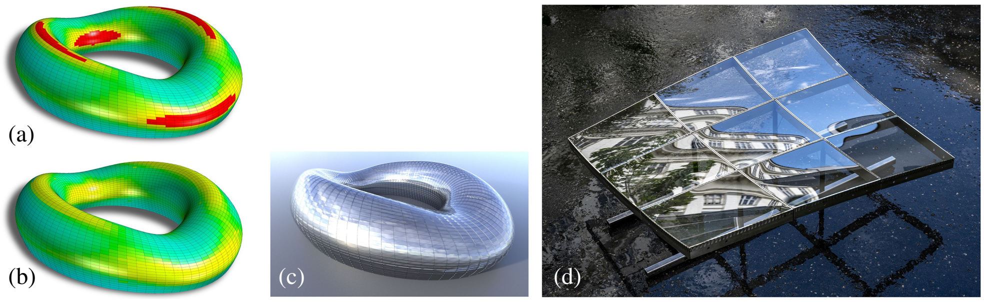

Figure 4: Here, we show the paneling with cold-bent glass of an initial design (a); the stress in the panels marked in red exceeds the design limit; these panels would break. After optimization (b), all panels can be built from planar cold bent glass. The rendering (c) uses the predicted shape of the panels, demonstrating a very smooth result. We verified our results by building a physical scale model (d) using borosilicate glass.

deformations (see Fig. 5) lies at the core of an algorithm for optimized paneling in this scenario [9]. The main idea is to use panels of constant Gaussian curvature K, since all panels of the same constant K can be formed on the same mold of that type. This mold is part of a sphere of radius $1/\sqrt{K}$ for $K > 0$ and part of any surface (e.g. a rotational one) with the right K for $K < 0$. To our big surprise, very few molds are necessary even for geometrically highly complex freeform designs (see Fig. 6). [8]. This method also uses developable surfaces ($K = 0$), which are obtained by bending flat sheets, and thus, no mold is required for those panels.

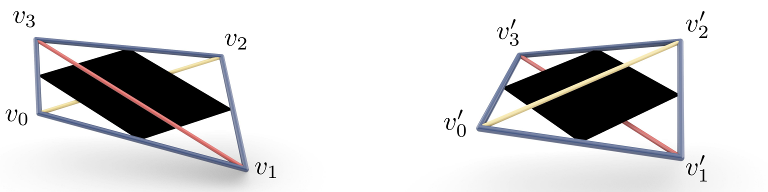

Figure 5: The computational model for isometric deformations uses quad meshes. In each quad, it considers the parallelogram (black) formed by the edge midpoints. It can be shown that an isometric deformation transforms these parallelograms in a rigid way. Equivalently, two quad meshes represent isometric surfaces, if for each pair of corresponding faces, diagonals v_0v_2, $v_0'v_2'$ and v_1v_3, $v_1'v_3'$ have the same length and enclose the same angle.

5 Fabrication-aware interactive computational design tools

Developable surfaces serve as illustrations for our next topic, which deals with interactive tools based on the latest results on computational design and optimization. Basically, all

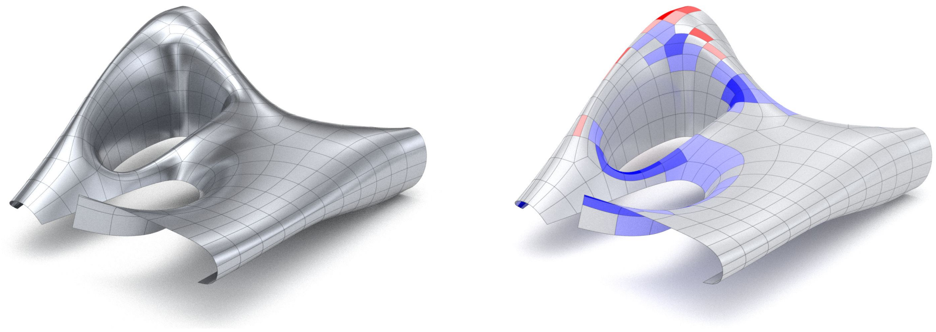

Figure 6: Digital model of the Heidar Aliyev Centre in Baku, Azerbaijan, designed by Zaha Hadid Architects. The original freeform design can be approximated with high accuracy using flexible panels from only 10 molds.

algorithms mentioned so far rely on nonlinear least squares formulations of a system of constraints that are preferably no more complicated than quadratic (see e.g. [8, 18]). Still, the underlying geometry representation matters a lot. Our latest work on developable surfaces [3] (Fig. 7) introduces a new definition for local *ruling* vectors, which are aligned with the direction in which one of the two principal curvatures vanishes, $\kappa_1 = 0$. This novel local criterion, which is independent of the parameterization of the surface, or its piecewise decomposition, enables interactive design without nonintuitive restrictions, something not achieved before. This developable editor is suitable for modeling fabrication techniques involving materials with minor stretching behavior, such as plywood or sheet metal (see Fig. 8). The careful formulation of constraints – here, developability –, together with the use of state-of-the-art parallel solvers, allows for design tools that are fast and efficient, and can be extended beyond to include further fabrication requirements, e.g. as in [2].

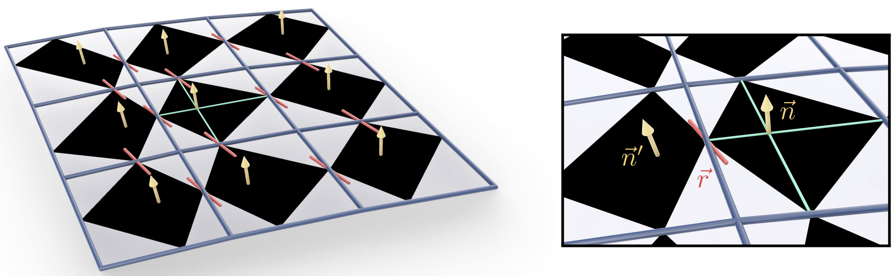

Figure 7: The computational model for developable meshes moves beyond inscribed parallelograms, allowing for any inscribed (black) planar quads. This, in turn, increases the expressive power for discrete developables. It is based on local *ruling* vectors $\vec{r}$, which are computed as the cross product of normals $\vec{n}, \vec{n}'$ of neighboring inscribed quads.

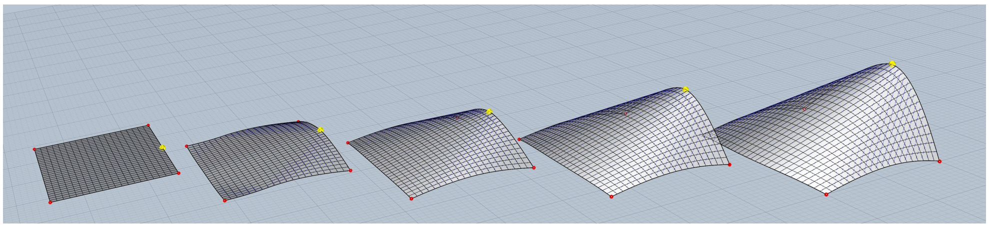

Figure 8: A sequence of still frames from an interactive design manipulation with developable surfaces. The introduction of new ad-hoc and efficient tools allows users to explore designs that could not be found with traditional methods.

5 Conclusion

Fundamental research in the field of architectural geometry is evolving quickly. New geometric insights yield to the development of novel interactive design tools and open up new, more efficient ways to build complex structures, for example, by exploiting material properties. It is interesting to note that, at the same time, the gap between research and practical application widens. Practitioners are often forced to use outdated tools, resulting in second-best or possibly even straight-out wrong results. We are currently using our insights in developable surfaces to develop an interactive modelling plugin for commonly used CAD systems. In the future, we will increase our efforts to bridge the gap to applications so that our research can better serve the demands of architects, designers, and engineers.

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