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Automatic Time Step Selection for Structural Dynamic Analysis of Train Crossing Events

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Abstract: In the design of railway bridges, dynamic effects induced by train passages can play a significant role. Numerical time integration methods enable efficient structural analyses in these cases. However, as this paper shows, suitable time step sizes are problem-dependent and difficult to assess. A remedy is provided by a newly developed automatic time step selection. It is presented here and illustrated with examples.

1 Structural Dynamic Analysis of Train Crossing Events

High-speed trains can excite bridges at resonance frequencies. Possible implications of the resulting large vibrations are a lower quality of the rail position and higher track maintenance costs. Therefore, if resonance effects cannot be excluded a-priori, the Eurocode requires bridges along high-speed railway lines to be verified based on a dynamic analysis [1].

There are two general modeling approaches for the dynamic analysis: The train can either be represented by a multi-body subsystem or by a series of axle loads moving with constant speed [1]. The multi-body approach takes the vehicle–bridge interaction into account, which reduces the structural response in most cases. However, the train properties must be known in detail and the computational cost is high when compared to the moving load model [2,3]. To consider the positive effect of the vehicle–bridge interaction in the commonly adopted moving load approach, the Eurocode provides additional damping values. It should be noted that they can lead to very different non-conservative results, whereas redesigned additional damping methods proved to be significantly more reliable [4].

Despite its higher efficiency, the moving load approach can still be computationally intensive because many different train types and speeds must be analyzed. The finite element method and numerical time integration schemes such as the Newmark method are commonly used to simulate each train crossing event. A key parameter for the computational costs here is the time step size. According to the Austrian Federal Railways (ÖBB) guideline [5], for example, it should be 1/20 of the oscillation period of the highest relevant eigenmode or shorter. This shows that the time steps usually need to be very small, around 1 ms or less, to ensure sufficient accuracy. However, if the higher eigenmodes do not have a significant contribution to the overall solution, a similar accuracy can be achieved with fewer time steps. Hence, the adequate time step size is problem-dependent and difficult to assess. Knowledge, experience, and multiple trials can help finding it [6].

This paper investigates the relationship between time step size and accuracy for different problem configurations in Section 2. Based on this study, a procedure to automatically determine an accurate and efficient time step size has been developed. It is presented in Section 3.

2 Time Step Size and Accuracy

The smaller the time step size, the higher is generally the accuracy. The precise relationship, however, is difficult to quantify because it depends on the physical and numerical parameters of the finite element model. To gain a better understanding of it, a parameter study was performed using the software SOFiSTiK [7]. After a brief description of the study, its results are discussed in this section.

2.1 Parameter Study

The test structures are two representative bridges. They are both simply supported and made from concrete but have different spans l of 15 m and 30 m. Their properties, listed in Table 1, were taken from Glatz and Fink [2] and are typical of concrete bridges. The specified damping ratio ζ is fulfilled at the first and third bending eigenfrequency using mass and stiffness proportional damping. Hence, the 15 m bridge has a mass proportional damping coefficient α of 1.73 and a stiffness proportional damping coefficient β of 7.14×10^{-5}, whereas the 30 m bridge has α set to 0.61 and β to 1.33×10^{-4}. Both test structures are subjected to load model HSLM-A5 moving with constant speed in the range from 140 km/h to 360 km/h. Furthermore, static initial conditions are considered.

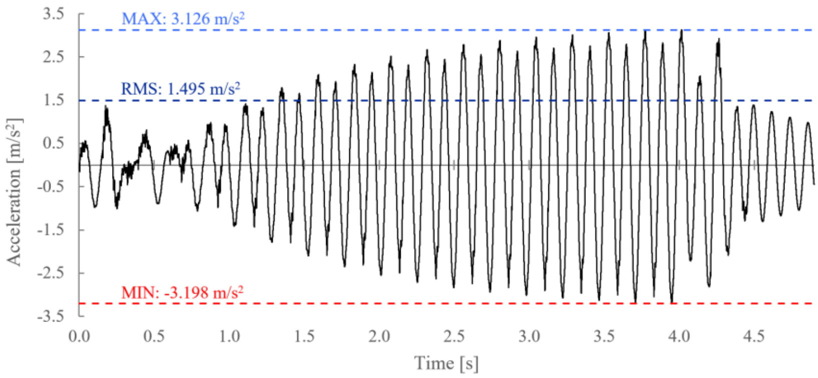
No.	Span l [m]	Mass per length μ [kg/m]	Bending stiffness EI [Nm^2]	Bending eigenfrequencies			Damping ratio ζ [%]
				f_1 [Hz]	f_2 [Hz]	f_3 [Hz]	
1	15	19042	2.667×10^{10}	8.26	33.01	74.15	1.85
2	30	29545	1.256×10^{11}	3.60	14.38	32.32	1.5

Table 1: Properties of the test bridges [2].

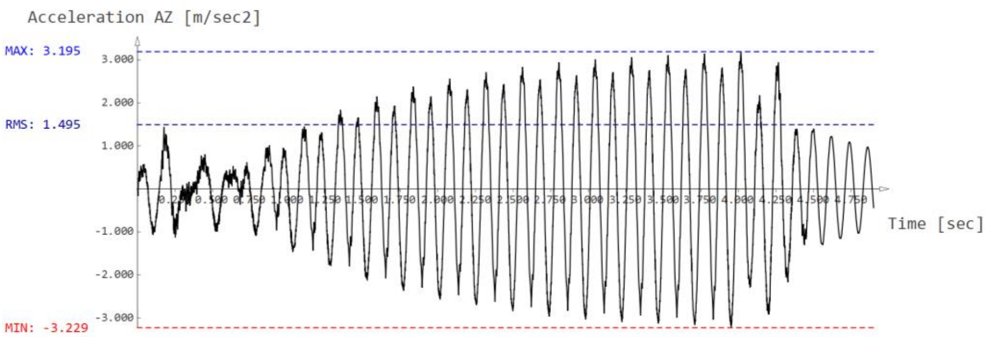
The finite element model of each bridge consists of 50 beam elements based on an enhanced Timoshenko approach with higher-order interpolation functions [7, 8]. The shear stiffness is chosen large enough to be in line with the Bernoulli theory used for the reference solution.

The Newmark constant acceleration method was employed to solve the equation of motion. Each train crossing event was computed multiple times using a smaller and smaller time step size. From one simulation to the next, it was reduced by a factor of 0.65. Among these different time step values is also the one specified in the ÖBB guideline [5]. It is 0.674 ms for the 15 m bridge and 1.547 ms for the 30 m bridge.

The accuracy of the numerical results is assessed by the relative error of the maximum absolute acceleration a_{max} at the center of the bridge.[1] To obtain the reference solution, the response of the ten lowest bending modes was calculated analytically and superimposed using the computer algebra system Maxima [9]. Figure 1 exemplarily shows the analytical and numerical acceleration of the 15 m bridge if the train speed is 327 km/h. As can be seen, the chosen speed is a critical one, inducing resonance of the first bending mode.



(a)



(b)

Figure 1: Acceleration at the center of the 15 m bridge under HSLM-A5 with a speed of 327 km/h: (a) analytical result, (b) numerical result using a time step size of 0.674 ms.

[1] The maximum absolute displacement u_{max} was also investigated in this study. It proved to be less sensitive to the variation of the time step size than the acceleration a_{max}. The discussion in this paper focuses on a_{max} for conciseness as it is the more relevant result in this context.

2.2 Discussion

The relative error of the numerical acceleration $a_{\max}$ is depicted in Figure 2 for different train speeds and time step sizes. The time step size required by the ÖBB guideline [5] is marked in bold. For both the 15 m bridge (Figure 2a) and the 30 m bridge (Figure 2b), it results in small relative errors of less than 6 % throughout all considered train speeds. A further reduction of the time step mostly leads to vanishing errors. Only a few slight errors remain, for example, in Figure 2b at speeds of 160 km/h and 340 km/h. Preliminary investigations indicate that these inaccuracies may arise from the spatial approximation of the moving loads by equivalent nodal loads. To improve the accuracy of the load representation, the number of nodes can be increased.

With larger time steps, high relative errors of more than 10 % occur at certain train speeds, presumably because the third bending mode is not represented well enough in time. Nevertheless, for other train speeds, good accuracy is obtained even for these "coarse" time steps. These include train speeds where the contribution of the first bending mode dominates the overall solution. The best examples are the design-relevant critical train speeds leading to a resonating first bending mode. They are marked in bold.

Time step size [ms] \ Train speed [km/h]	140	**164**	180	200	**218**	240	260	280	300	**327**	340	360
3.778	8.3	6.5	43.0	13.3	4.8	7.0	7.9	6.5	17.6	1.4	3.3	12.1
2.455	9.5	2.1	3.8	2.2	3.9	5.6	4.6	4.1	2.5	1.8	2.1	13.4
1.596	6.7	5.5	2.3	0.1	1.8	0.1	10.8	7.5	6.6	1.4	1.1	1.4
1.037	5.9	2.3	3.6	2.6	1.4	0.6	4.5	1.6	5.6	1.1	1.6	2.2
0.674	0.9	3.4	1.7	4.4	2.4	5.3	2.7	2.9	4.4	1.0	0.3	1.6
0.438	0.9	2.7	3.1	3.4	0.9	1.4	1.6	2.0	2.5	0.7	0.3	2.0
Reference solution [m/s²]	*0.50*	*0.74*	*0.49*	*1.06*	*2.47*	*1.35*	*1.17*	*1.16*	*1.27*	***3.20***	*2.51*	*2.40*

(a)

Time step size [ms] \ Train speed [km/h]	**143**	160	180	200	220	240	260	**285**	300	320	340	360
8.666	2.1	29.7	0.8	0.9	6.9	2.4	0.6	4.9	7.6	12.7	6.5	15.2
5.633	2.4	25.9	8.4	7.3	2.7	2.8	3.5	3.5	2.7	6.6	4.4	3.8
3.662	1.9	9.2	1.5	10.0	3.8	10.9	1.4	3.4	0.9	16.2	0.7	7.7
2.380	1.6	0.2	4.3	1.3	1.9	6.4	1.1	1.1	0.5	1.2	0.5	4.0
1.547	0.0	5.3	1.7	3.2	0.3	4.0	5.6	0.6	0.2	3.0	4.5	1.4
1.006	0.2	6.6	0.6	3.0	0.1	0.7	0.1	0.5	0.5	5.2	6.3	2.8
Reference solution [m/s²]	*0.81*	*0.23*	*0.34*	*0.44*	*0.54*	*0.52*	*0.65*	***2.31***	*1.29*	*0.90*	*0.65*	*0.66*

(b)

Figure 2: Relative error [%] of the numerical acceleration $a_{\max}$ for different train speeds and time step sizes: (a) 15 m bridge, (b) 30 m bridge.

This study illustrates that the error encountered in train passage simulations can significantly vary depending on different parameters such as the train speed. Therefore, time step sizes given in the literature may only serve as a rough estimate. They can still lead to unnecessarily high computational costs or insufficient accuracy. In the following section, an automatic and problem-based procedure is presented to determine an accurate yet efficient time step size.

3 Automatic Time Step Selection

An established way to detect the time step size that is needed to avoid considerable time discretization errors is a convergence study. In this iterative procedure, the result-time curve is computed multiple times using a smaller and smaller time step size until the result changes only marginally.

The start value of the time step size used in the convergence study must be chosen carefully. If it is too large, the results do not always converge strictly monotonically, as depicted in Figure 2. A major reason for this behavior is that vibrations can be significantly over- or underestimated unless they are resolved by enough time steps. In the worst case, convergence is declared too early if the contribution of a relevant bending mode is underestimated. For example, this effect can be seen in Figure 2a at speeds of 140 km/h and 360 km/h. Thus, the convergence study primarily serves in this context to verify a time step size rather than to find it.

A time step size suitable for this verification is the one defined by [5]. In this work, it proved to accurately resolve all relevant eigenmodes of the test bridges. However, it is quite small. Applying it to compute not only nodal results but also all design-relevant internal forces and moments can be time-consuming. These computational costs are not always necessary. Figure 2 shows that in many cases low relative errors can also be obtained with considerably larger time steps.

Therefore, the proposed method to select the time step size consists of two steps:

1. A convergence study is performed to verify a time step size that is small enough to resolve all relevant eigenmodes in time. Such a verification is also specified in the ÖBB guideline [5]. It states that a 0.65 times reduction of the time step size should not change the maximum result by more than 10 %. The value of the time step size that is to be verified can be easily determined based on the specifications in [5] or other technical codes.
2. Larger time step sizes within an appropriate range are checked to see if they lead to results of similar accuracy. To avoid coincidental agreement, the root-mean-square of the complete result-time curve should be considered here in addition to the maximum absolute value.[2]

This approach can be largely automated, ensures accurate results, and allows the design-relevant internal forces and moments to be computed efficiently. Furthermore, the

[2] The root-mean-square of the result-time curve was also determined in the parameter study described in Section 2 as an additional check of accuracy.

computational cost of this preparatory procedure is relatively small because only primary results (nodal results) are calculated.

Tables 2 and 3 show the time step sizes resulting from the proposed method when applied to the test bridges using a tolerance of 5 %. Compared to the standard approach, the number of time steps is reduced significantly, on average, by a factor of 4.0 for the 15 m bridge and 2.5 for the 30 m bridge. A further reduction of the number of time steps could possibly be achieved by extending the range of time step sizes considered into areas where the result quality fluctuates. The reliability of this extension is subject to further investigation. Preliminary results indicate feasibility.

Train speed [km/h]											
140	164	180	200	218	240	260	280	300	327	340	360
0.674	2.455	2.455	2.455	5.812	2.455	1.037	1.037	2.455	5.812[3]	3.778	1.596

Table 2: Time step sizes [ms] resulting from the proposed method for the 15 m bridge.

Train speed [km/h]											
143	160	180	200	220	240	260	285	300	320	340	360
8.666[3]	1.547	3.662	2.380	2.380[3]	1.547	1.006	13.333	5.633	1.547	1.547	3.662

Table 3: Time step sizes [ms] resulting from the proposed method for the 30 m bridge.

4 Conclusions

The relationship between time step size and accuracy in train passage simulations was investigated. For this purpose, a parameter study was performed on two test bridges by varying the time step size and train speed. Furthermore, the numerical results were compared with analytical reference solutions. The study confirms the suitability of the time step size specified in [5]. In the case of both bridges, it leads to accurate numerical results throughout all considered train speeds. In many cases, however, low relative errors are also obtained with considerably larger time steps. Based on these findings, a method to automatically determine an accurate and efficient time step size was developed. Moreover, it was applied to the test bridges, leading to a significant reduction of the number of time steps when compared to the standard approach.

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